MATHEMATICS

1. If slope of tangent at any point p(x, y) on the curve y = f(x) is $\frac{x^2+y^2}{2\times y}$ and y(1) = 2, then find y(8)

Ans. $6\sqrt{2}$

Sol. $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

$2xy\,dy = x^2\,dx + y^2\,dy$

$2xy\,dy - y^2\,dx = x^2 dx$

$d\left(\frac{y^2}{x}\right)=dx$

$\Rightarrow \frac{y^2}{x}=x+C$

$\Rightarrow y^2 = x(x + c)$

$\Rightarrow y^2 = x(x + c)$

$y=\pm\sqrt{x(x+C)}$

$\therefore$ y(1) = 1 $\Rightarrow$ C = 1

$y=\sqrt{x(x+1)}$

$y(8)=\sqrt{8\times 9}=6\sqrt{2}$

2. Coefficient of x^7 in $\left(1-x+x^3\right)^{10}$

Ans. $\frac{10!}{4!5!}-\frac{10!}{3!7!}-\frac{10!}{2!7!}$

Sol. Coefficient x^7 in $(1 - x + x^3)^{10}$

$\frac{10!}{a!b!c!}(1)^a(-x)^b(x^3)^c$

a + b + c = 10

b + 3c = 7

c	b	a
0	7	3
1	4	5
2	1	7

.......(1)

$=-\frac{10!}{3!7!0!}+\frac{10!}{1!4!5!}-\frac{10!}{2!1!7!}$

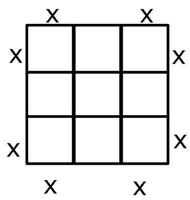
$\Rightarrow 10!\left(\frac{1}{4!5!}-\frac{1}{3!7!}-\frac{1}{2!7!}\right)$

$\Rightarrow \frac{10!}{4!5!}-\frac{10!}{3!7!}-\frac{10!}{2!7!}$

3. There is a sheet of dimension 30cm × 30cm, and if we make an open box with maximum volume using this sheet, then find the surface area of this box.

Ans. 800

Sol.



$v = (30-2x)^2 \times x$

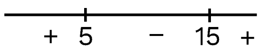
$$\frac{dy}{dx} = (30-2x)^2 + x.2(30-2x)(-2)$$

$$\frac{dv}{dx} = (30-2x)\{30-2x-4x\}$$

$$\frac{dv}{dx} = (30-2x)\{30-6x\}$$

$= (2x-30)(6x-30)$

For volume max, $\frac{dv}{dx} = 0$



$\Rightarrow (30-2x)(30-6x) = 0 \Rightarrow x = 15, 5$

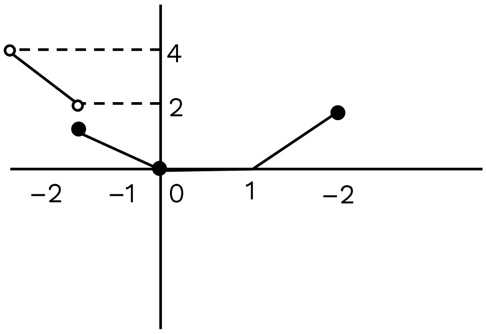
For max (x = 5)

So, surface Area = 400 + 4 × 100 = 800

4. If $f(x) - \begin{cases} x[x], & -2 < x < 0 \\ (x-1)[x], & 0 < x < 2 \end{cases}$ and "m' are the points of discontinuity and "n" are the points of non-differentiability then find m + n?

Ans. 4

Sol.



m = 1 , n = 3

m + n = 4

5. Find the total number of values of $n \in Z$, given that $|n^2 - 10n + 19| < 6$.

Ans. 7

Sol. $-6 < x^2 - 10x + 19 < 6$

Case-(I) $\Rightarrow x^2 - 10x + 25 > 0$

$(x-5)^2 > 0$

Case-(II) $\Rightarrow x^2 - 10x + 13 > 0$

$$x = \frac{10 \pm \sqrt{100-52}}{2}$$

$= \dfrac{10 \pm \sqrt{40}}{2}$

$= \dfrac{10 \pm 4\sqrt{3}}{2}$

$5 - 2\sqrt{3} < x < 5 + 2\sqrt{3}$

6. Equation of an ellipse is $x^2 + 9y^2 = 9$, which cuts the positive x and positive y-axis at A & B. Suppose P is any point on auxiliary circle Find maximum area of triangle PAB.

Ans. $\frac{1}{2}\left(3 + 3\sqrt{10}\right)$

Sol. Area $= \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 3\cos\theta & 3\sin\theta & 1 \end{vmatrix}$

$= \frac{1}{2}\left(3 - 3\cos\theta - 9\sin\theta\right)$

Maximum Area $= \frac{1}{2}\left(3 + 3\sqrt{10}\right)$

7. There is a set of numbers {1, 2, 3, 4, 5, 6, 7} then find how many numbers are formed such that three numbers {1, 2, 4} are not together as well as {3, 5, 6, 7} are also not together.

Ans. 4680

Sol. $7! - 3! \times 4! \times 2!$

$\Rightarrow 7 \times 6 \times 5 \times 4! \;\; - 3! \times 2! \times 4!$

$\Rightarrow 4!\,[210 - 12]$

$\Rightarrow 4! \times 198 = 198 \times 4! = 4752$

8. If $x^2f(x) - x = 4\int_0^x t\,f(t)\,dt$ & $f(1) = \frac{2}{3}$. Find 18f(3)

Ans. 160

Sol. $x^2f(x) - x = 4\int_0^x t.f(t)dt$

$x^2f'(x) + 2x\,f(x) - 1 = 4x.f(x)$

$\Rightarrow x^2\dfrac{dy}{dx} = 2xy + 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2xy+1}{x^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2y}{x} + \dfrac{1}{x^2}$

$\Rightarrow \dfrac{dy}{dx} - 2y\left(\dfrac{1}{x}\right) = \dfrac{1}{x^2}$ (Linear D.E)

$\text{I.F} = e^{\int \frac{-2}{x}dx} = e^{-2\ln x} = \dfrac{1}{x^2}$

Solution of D.E

$$y \times \frac{1}{x^2} = \int \frac{1}{x^4} dx$$

$$\Rightarrow \frac{y^2}{x^2} = \frac{-1}{3x^3} + C$$

$$\Rightarrow \frac{2}{3} = \frac{-1}{3} + C \Rightarrow C = 1$$

So, $\frac{y}{x^2} = \frac{1}{3x^3} + 1 \Rightarrow \frac{y}{9} = \frac{-1}{81} + 1$

$$\Rightarrow y = 9\left[\frac{80}{81}\right] = \frac{80}{9}, \ 18y = 160$$

9. Given that $\frac{2z+2i}{2z-i}$ is purely imaginary. Such that z = x + iy and $x^2 + y = 0$ then find $y^2 + y + 1 = ?$

Ans. $\frac{3}{4}$

Sol. Let $w = \frac{2z+2i}{2z-i}$ in purely imaginary.

$$\Rightarrow w = -\overline{w} \Rightarrow \frac{2z+2i}{2z-i} = \frac{2\overline{z}-2i}{2\overline{z}+i}$$

$$\Rightarrow (2z+2i)(2\overline{z}+i) = (2z-i)(-2\overline{z}+2i)$$

$$\Rightarrow 4|z|^2 + 2iz + 4i\overline{z} - 2 = -4|z|^2 + 4iz + 2i\overline{z} + 2$$

$$\Rightarrow 8|z|^2 - 2iz + 2i\overline{z} - 4 = 0$$

$$\Rightarrow \frac{m}{n}$$

$\Rightarrow 8(x^2 + y^2) + 4y - 4 = 0$

$2x^2 + 2y^2 + y - 1 = 0$

$\Rightarrow -2y + 2y^2 + y - 1 = 0$

$\Rightarrow 2y^2 - y - 1 = 0$

$y = 1$ or $y = -\frac{1}{2}$ Accepted

10. Find the value of $96\cos\left(\frac{\pi}{33}\right)\cos\left(\frac{2\pi}{33}\right)\cos\left(\frac{4\pi}{33}\right)\cos\left(\frac{8\pi}{33}\right)\cos\left(\frac{16\pi}{33}\right) = ?$

Ans. $\frac{1}{32}$

Sol. let $\theta = \frac{\pi}{3^3}$ then

$y = \cos\theta. \cos2\theta. \cos2^2\theta. \cos2^3\theta. \cos2^4\theta$

$$y = \frac{\sin 2^5\theta}{2^5 \sin\theta} = \frac{\sin 32\theta}{32\sin\theta}$$

$$y = \frac{\sin\left(\frac{32\pi}{33}\right)}{32\sin\left(\frac{\pi}{33}\right)} = \frac{\sin\left(\pi - \frac{\pi}{33}\right)}{32\sin\left(\frac{\pi}{33}\right)} = \frac{11}{32}$$

96y = 3

$$y = \frac{3}{96} = \frac{1}{32}$$

11. If the coefficients of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal then find $a^4 \cdot b^4$ = ?

Ans. $\frac{{}^{13}C_6}{{}^{13}C2}$

Sol. Coefficient $x^7 \Rightarrow {}^{13}C_r\,(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$${}^{3}C_r a^{13-r}\left(-\frac{1}{b}\right)^r x^{13-3r}$$

$13 - 3r = 7 \Rightarrow r = 2$

Coefficient $x^{-5} \Rightarrow {}^{13}C_r\,(ax)^{13-r}\left(\frac{1}{bx^2}\right)^r$

$13 - 3r = -5 \Rightarrow 3r \pm 5 \quad r = 6$

$${}^{13}C_2\, a^{11}\left(-\frac{1}{b}\right)^2 = {}^{13}C_6\, a^7\,\frac{1}{b^6}$$

$$a^4 b^4 = \frac{{}^{13}C_6}{{}^{13}C_2}$$

12. In an AP 3, 8......,373 find the sum of those term which are not divisible by 3.

Ans. 9525

Sol. $\Rightarrow a = 3, \qquad d = 5$

No. which are divisible by 3 in this AP are $\Rightarrow$ 3, 3 + 5 × 3, 3 + 5 × 6,........3 + 5 × 72.

Their sum = (3) + (3 + 5.3) + (3 + 5. 6)+.........(3 + 5. 72)

= 3. (25) + 5(3 + 6+..................72)

= 3. (25) + 5. 3(1 + 2+..................24)

$$= 75 + 15\frac{(24 \times 25)}{2}$$

= 4575

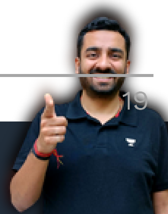
Sum of All numbers = 3 + 8 + ...373

$$= \frac{75}{2}(3 + 373)$$

$$= \frac{75}{2}(376) = 14100$$

Hence, required Ans = 14100 – 4575 = 9525

13. Find Perpendicular distance of point of intersection of line $\frac{x-3}{-2} = \frac{y-5}{3} = \frac{z-1}{5}$ and plane x + y + z = 12 to the plane 2x +5y + 7z = 32



Ans. $\frac{29}{\sqrt{78}}$

Sol. let point is $(-2\lambda + 3, 3\lambda + 5, 5\lambda + 1)$

$\Rightarrow 6\lambda + 9 = 12$

$\Rightarrow \lambda = \frac{1}{2}$

$\Rightarrow$ point is $\left(2, \frac{13}{2}, \frac{7}{2}\right)$

Distance $= \left|\frac{4 + \frac{65}{2} + \frac{49}{2} - 32}{\sqrt{78}}\right|$

$= \frac{29}{\sqrt{78}}$

14. Class intervals and their frequencies are given as follows:

2–10	10–18	18–24	24–32
1	12	10	x

and Mean for the distribution is 18 then find variance?

Ans. 2280.31

Sol.

Class	x_i	f_i	$x_i f_i$	$(x_i f_i)^2$
2–10	6	1	6	36
10–18	14	12	168	21224
18–24	21	10	210	44100
24–32	28	x	28x	784x

Mean $= \frac{\sum x_i f_i}{\sum f_i}$

$\Rightarrow 18 = \frac{6 + 168 + 210 + 28x}{23 + x}$

$\Rightarrow 18 \times 23 + 18x = 384 + 28x \Rightarrow 10x = 30 \Rightarrow x = 3$

$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{f_i}\right)^2$

$\sigma^2 = \frac{36 + 21224 + 44100 + 784 \times 3}{1 + 12 + 10 + 3} - 18^2$

$\sigma^2 = \frac{67712}{26} - 324 = 2604.3076 - 324$

$\Rightarrow \sigma^2 = 2280.31$

15. Integrate $I = \int e^{\sin^2(x)}\left(\sin 2x \cos x - \sin(x)\right)dx$

Ans. $e^{\sin^2(x)} \cos x + c$

Sol. $I = \int e^{\sin^2(x)} \sin(2x)\left(\cos x - \frac{1}{2\cos(x)}\right) dx$

Let $\sin^2(x) = t$

So, sin 2x dx = dt

So, $I = \int e^t\left(\sqrt{1-t} - \frac{1}{2\sqrt{1-t}}\right) dt$

Hence, $\int e^t\left(f(t) + f'(t)\right) dt = e^t f(t) + c$

$I = e^t\sqrt{1-t} + c = e^{\sin^2(x)}\cos(x) + c$

16. Two dice are rolled and sum of numbers of two dice is N then probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then 11m – 3n is?

Ans. 85

Sol. All possible sums = {2, 3, 4, ..., 12}

But for $N! > 2^N$ we must have $n \geq 4$

So, favorable sums = {4, 5, ..., 12}

Required probability $= 1 - P(S_2 \cup S_3)$

Where, $P(S_2 \cup S_3)$ = probability of sum either 2 or 3

$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \times 2$

$= \frac{1}{12}$

Hence, required probability $= 1 - \frac{1}{12} = \frac{11}{12}$

So, m = 11 & n = 12

Hence, 11 × m – 3 × n = 85

17. If the number of ways in which a mixed double badminton can be played such that no couples played into a same game is 840. Then find the number of players.

Ans. 16

Sol. H_1 H_2 H_3 H_4..........H_n

W_1 W_2 W_3 W_4..........W_n

So, ${}^nC_2 . {}^{n-2}C_2 \times 2 = 840$

$\Rightarrow \frac{n(n-1)}{2} . \frac{(n-2)(n-3)}{2} = 420$

$\Rightarrow$ n(n–1) (n–2) (n–3) = 5 × 6 × 7 × 8

So, n = 8

Total number of players = 2n = 16

18. Find the shortest distance between the lines: $\frac{x-2}{1} = \frac{y}{2} = \frac{z-1}{0}$; $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{2}$

Ans. 1

Sol. Distance $= \dfrac{\begin{vmatrix} 0 & -3 & 0 \\ 1 & 2 & 0 \\ 1 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & -2 & 2 \end{vmatrix}} = \dfrac{6}{\sqrt{4^2 + z^2 + 4^2}}$

$= \dfrac{6}{\sqrt{36}}$

$= 1$