MATHEMATICS

1. Number of ways to form 4 letter word from letters of the word "UNIVERSE", such that it has 2 vowels and 2 consonant :

(A) 504 (B) 302 (C) 106 (D) 504

Ans. (A)

Sol. Vowels : U, I, E, E

Consonant : N, V, R, S

$$\left[\text{Vowels} \ - \begin{array}{l}[2\,\text{alike} \to 1]\\ [2\,\text{different} \to {}^3C_2 . 2^1 = 6]\end{array}\right]$$

So, number of words = ${}^4C_2 \times 12 \times 7 = 504$

2. Find the rank of word "PUBLIC" in dictionary.

(A) 581 (B) 582 (C) 580 (D) 583

Ans. (B)

Sol. B, C, I, L, P, U

Rank = $\left(4.\underline{|5} + 4\underline{|4} + 2\underline{|2} + 2\right) = 582$

3. Given that f(x) + f(π – x) = π^2, find the value of $\int_0^{\pi} f(x)\sin x\,dx$.

(A) π^2 (B) $2\pi^2$ (C) $\pi^2/3$ (D) 0

Ans. (A)

Sol. $I = \int_0^{\pi} f(x)\sin x\,dx$

Apply Property:

$I = \int_0^{\pi} f(\pi - x)\sin(\pi - x)\,dx$

$= \int_0^{\pi} \left(\pi^2 - f(x)\right)\sin x\,dx$

$I = \pi^2 \int_0^{\pi} \sin x\,dx - I$

$2I = 2\pi^2$

$I = \pi^2$

4. Given that $1^2 - 2^2 + 3^2 - 4^2 + - - - - + 2023^2 = (1012)m^2 n$ and (gcd)(m, n) = 1 then find the value of $m^2 - n^2$.

Ans. 240

Sol. –(1 + 2 + 3+.....+2022) + $(2023)^2$

$(2023)^2 - \frac{(2022)(2023)}{2}$

(2023) (1012) = $(1012)m^2n$

$\Rightarrow$ (1012) (7) $(17)^2$ = (1012) $m^2(n)$

$\therefore$ m = 17

n = 7

$\therefore$ $m^2 - n^2 = 240$

5. If Probability of throwing three dice such that each dice has different outcome is $\frac{p}{q}$. Then value of $q - p$ is:

(A) 4 (B) 3 (C) 5 (D) 6

Ans. (A)

Sol. Probability = $\frac{{}^6C_3 \lfloor 3}{6^3} = \frac{5}{9} = \frac{p}{q}$

$\therefore q - p = 4$

6. There are 100 students, 70 of them are good in English and 55 are good in Hindi. α students are good in English only and β students are good in Hindi only. Then find the eccentricity of $25\beta^2x^2 + \alpha^2y^2 = \alpha^2\beta^2$:

(A) $\frac{\sqrt{91}}{10}$ (B) $\frac{\sqrt{91}}{11}$ (C) $\frac{\sqrt{92}}{10}$ (D) $\frac{\sqrt{92}}{11}$

Ans. (A)

Sol. English (A), Hindi (B)

$n(A) = 70$, $n(A \cap B) = 100 = 70 + 55 - n(A \cap B)$

$n(B) = 55$ $n(A \cap B) = 25$

$n(A) - n(A \cap B) = \alpha$ $\therefore \alpha = 45$

$n(B) - n(A \cap B) = \beta$ $\therefore \beta = 30$

ellipse $\frac{x^2}{\left(\frac{\alpha}{5}\right)^2} + \frac{y^2}{\beta^2} = 1$

$\frac{x^2}{9^2} + \frac{y^2}{30^2} = 1$

$\therefore 9^2 = 30^2(1-e^2)$

$e^2 = \frac{91}{100}$

$e = \frac{\sqrt{91}}{10}$

7. Given that coefficient of x^7 in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ = A and the coefficient of x^{-7} in $\left(ax + \frac{1}{3bx^2}\right)^{11} = B$. then choose the correct options If A = B.

(A) 729a = 32b (B) 32a = 729b

(C) 32 × 729 = ab (D) 32 = 729 ab

Ans. (D)

Sol. $\left(ax^2 + \frac{1}{2bx}\right)^{11} \to x^7$

$T_{r+1} = {}^{11}C_r \left(ax^2\right)^{11-r} \left(\frac{1}{2bx}\right)^r$

$T_{r+1} = {}^{11}C_r a^{11-r} \frac{1}{(2b)^r} x^{22-3r}$

$\Rightarrow 22 - 3r = 7 \Rightarrow 3r = 15$

$\boxed{r = 5}$

$A = \boxed{{}^{11}C_5 - a^6 \dfrac{1}{2^5 b^5}}$

$T_{r+1} = {}^{11}C_r + 1(an)^{11-r}\left(\dfrac{1}{3bx^2}\right)^r$

$T_{r+1} = {}^{11}C_r a^{11-r} \dfrac{1}{(3b)^r} x^{11-3r}$

$11-3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$B = {}^{11}C_6 a^5 \dfrac{1}{3^6 b^6}$

When A = B

${}^{11}C_5 - a^6 \dfrac{1}{32 \times b^5} = {}^{11}C_6 a^5 \dfrac{1}{3^6 b^6}$

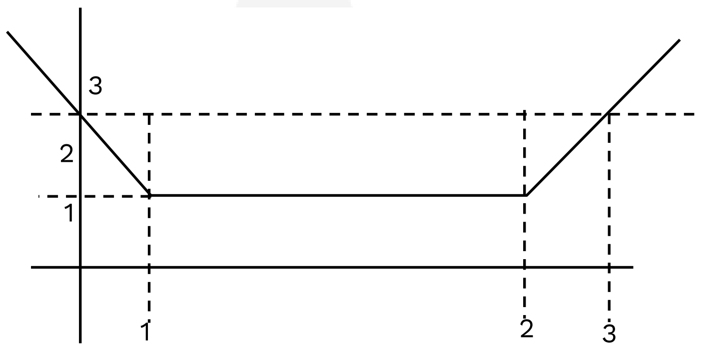
$\Rightarrow \dfrac{a}{32} = \dfrac{1}{3^6 b}$

$\Rightarrow \{32 = 729\ ab\}$

8. Find the area under the curve y = |x − 1| + |x − 2| and y = 3.

Ans. 4

Sol.



$A = \dfrac{1}{2} \times 2 \times 1 + 1 \times 2 + \dfrac{1}{2} \times 1 \times 2$

A = 1 + 2 + 1 = 4

9. Sum of all values of α for which $\hat{i} - 2\hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{j} + 4\hat{k}$, $(\alpha+1)\hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha-8)\hat{j} + 6\hat{k}$ are coplanar is:

(A) 2 (B) 4 (C) −2 (D) −4

Ans. 2

Sol. let A(1, −2, 3), B(2, −3, 4), C(α+1, 0, 2)

and D(9, α−8, 6)

$\overrightarrow{AB} = \hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{AC} = \alpha\hat{i} + 2\hat{j} - \hat{k}$

$\overrightarrow{AD} = 8\hat{i} + (\alpha - 6)\hat{j} + 3\hat{k}$

$\therefore \left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix} = 0$$

$1(6 + \alpha - 6) + 1(3\alpha + 8) + 1(\alpha^2 - 6\alpha - 16) = 0$

$\alpha^2 - 2\alpha - 8 = 0$

$(\alpha - 1)^2 = 9 \Rightarrow \alpha = 4, -2$

Sum = 2

10. Find rank of the word "MOTHER" in the dictionary.

(A) 310 (B) 309 (C) 308 (D) 311

Ans. (B)

Sol.

3 4 6 2 1 5
M O T H E R
2 2 3 1 0 0

$5I_0\ 4I_0\ 3I_0\ 2I_0\ 1I_0\ 0I_0$

$5I_0 \times 2 + 4I_0 \times 2 + 3 \times 3_0 + 1 \times 2I_0$

$\Rightarrow 240 + 48 + 10 + 2$

$\Rightarrow 260 + 48 = 308 + 1 = 309$

11. Find the value of tan9° – tan27° – tan63° + tan81° :

(A) 5 (B) 4 (C) 3 (D) 2

Ans. (B)

Sol. $(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} \qquad \{\tan\theta + \cot\theta = 2\,\mathrm{cosec}\,2\theta\}$$

$$= \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1} = \frac{8\left(\sqrt{5}+1-\sqrt{5}+1\right)}{4}$$

$= 4$

12. Find the square of distance of point (12, 12, 18) from a plane which passes through line of intersection of planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 6$ and $\vec{r}.\left(2\hat{i}+3\hat{j}+4\hat{k}\right) = -5$ and also passes through point (0, 2, –2).

Ans. 620

Sol. $x + y + z = 6$

$2x + 3y + 4z = -5$

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$

$\Rightarrow (2-2) - 6) + \lambda\,(0 + 6 - 8 + 5) = 0$

$-6 + 3\lambda = 0 \qquad (\lambda = 2)$

$(x + y + z - 6) + 4x + 6y + 8z + 10 = 0$

$\Rightarrow 5x + 7y + 9z + 4 = 0$

$$\Rightarrow \text{Distance} = \left|\frac{5\times 12 + 7\times 2 + 9\times 18 + 4}{\sqrt{25+49+81}}\right| = \frac{310}{\sqrt{155}}$$

$$\text{Answer} = \left(\frac{310}{\sqrt{155}}\right)^2 = 620$$

13. Given that

$20^{19} + 2.21.(20)^{18} + 3(21)^2(20)^{17} + 4(21)^3(20)^{16} + \ldots + 20(21)^{19} = S$

Find the value of S:

Ans. $(20)^{21}$

Sol. $S = 20^{19} + 2.21(20)^{18} + 3(21)^2\,(20)^{17} + \ldots\ldots + 20(21)^{19}$

$$\frac{S}{(21)^{19}} = \left(\frac{20}{21}\right)^{19} + 2\left(\frac{20}{21}\right)^{18} \times 3\left(\frac{20}{21}\right)^{17} + \ldots + 20$$

Let $\frac{S}{(21)^{19}} = k$

$$\text{So, } k = \left(\frac{20}{21}\right)^{19} + 2.\left(\frac{20}{21}\right)^{18} + 3\left(\frac{20}{21}\right) + \ldots.20$$

$$\frac{20k}{21} = 20\left(\frac{20}{21}\right) + 19\left(\frac{20}{21}\right)^2 + \ldots. + 2\left(\frac{20}{21}\right)^{19} + \left(\frac{20}{21}\right)^{20}$$

$$\frac{k}{21} = 20 - \left(\frac{20}{21}\right) - \left(\frac{20}{21}\right)^2 + \ldots. - \left(\frac{20}{21}\right)^{20}$$

$$\Rightarrow \frac{k}{21} = 20 - \left[\frac{20}{21}\left[\frac{\left(1 - \left(\frac{20}{21}\right)^{20}\right)}{1 - \frac{20}{21}}\right]\right] = \frac{k}{21} = \frac{20(20)^{20}}{(21)^{20}}$$

$$\Rightarrow \frac{S}{(21)^{19} \times 21} = \frac{20^{21}}{21^{20}} \Rightarrow S = (20)^{21}$$

14. Check the statements

Statement: 1 $(p \Rightarrow q) \vee (\sim p \wedge q)$: tautology

Statement: 2 $(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$: contradiction

(A) Both are true
(B) Neither 1 nor 2 are true
(C) Only First One is true
(D) Both are true

Ans. (B)

Sol. $(\sim p \vee q) \vee (\sim p \wedge q)$

$\equiv \sim p \vee q$

$\sim(\sim q \vee p) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge q$.

Hence neither is true.

15. Check the statements

S_1: $(2023)^{2022} - (1999)^{2022}$ is divisible by 8

S_2: $13(13)^n - 11n - 13$ is divisible by 144 for infinite values of $n \in N$.

(A) S_1 & S_2 both are correct
(B) S_1 & S_2 both are incorrect
(C) S_1 is correct S_2 is incorrect
(D) S_1 is incorrect & S_2 is correct

Ans. (A)

Sol. S_1 : Note that $a^n - b^n$ is divisible by $a-b$

Hence $(2023)^{2022} - (1999)^{2022}$ is divisible by 24 hence by 8

S_2 : $13(12 + 1)^n - 11n - 13$

$$= 13\left[{}^nC_0 12^n + \ldots + {}^nC_{n-2} 12^2 + {}^nC_1 12 + 1\right] - 11n - 13$$

$= 144k + (156-11)n$

$= 144k + 145n$

Hence if n is a multiple 144 it is divisible by 144
So infinite such n exists.

16. The system of equations.
P_1: x + y + z = 6
P_2: x + 2y + αz = 5
P_3: x + 2y + 6z = β has
(A) Infinitely many solutions α = 6, β = 3
(B) Infinitely many solutions α = 6, β = 5
(C) Unique's solutions α = 6, β = 5
(D) No solutions α = 6, β = 5

Ans. (B)

Sol. $$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & \alpha \\ 1 & 2 & 6 \end{vmatrix},\ \Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & \alpha \\ \beta & 2 & 6 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & \alpha - 1 \\ 1 & 1 & 6-1 \end{vmatrix}\ \Delta_1 = \begin{vmatrix} 0 & 0 & 1 \\ -7 & 2-\alpha & \alpha \\ \beta - 12 & -4 & 6 \end{vmatrix}$$

$\Delta = 5 - \alpha + 1$ $\qquad$ $\Delta_1 = 1[28-(2-\alpha)(\beta-12)]$
$\Delta = 6 - \alpha$ $\qquad$ $\Delta_1 = 28 + (\alpha-2)(\beta-12)$
For infinite many solution $\Delta = 0$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$
$\Delta = 0$
$\alpha = 6$
When $\Delta_1 = 0, \Rightarrow 28 + (6-2)(\beta - 12) = 0$
$\Rightarrow 4(\beta - 12) = -28$
$\Rightarrow \beta = 12 - 7 = 5$

$$\text{Now } \Delta_2 = \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 1 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$$

Hence, at α = 6 & β = 5
$\Delta = 0$, & $\Delta_1 = \Delta_2 = \Delta_3 = 0$
Thus at α = 6 & β = 5 system of equation has infinite solutions.

17. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

18. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

19. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

20. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

21. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

22. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

23. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

24. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

25. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

26. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

27. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

28. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

29. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

30. R

(A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.