MATHEMATICS

1. Pairs of dice is thrown 5 times, if sum 5 is considered success than find probability of getting 4 success:

Ans. $\frac{40}{95}$

Sol. favour outcomes $\{(1,4),(4,1),(2,3),(3,2)\}$

Success probability $= \frac{4}{36} = \frac{1}{9}$

Probability of getting 4 success$= {}^5C_4\left(\frac{1}{9}\right)^4 \frac{8}{9} = \frac{5\times8}{95} = \frac{40}{95}$

2. Find coefficient of x^{18} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

Ans. ${}^{15}C_6$

Sol. $T_{r+1} = {}^{15}C_r\, x^{60-4r} \times (-1)^r \times x^{-3r}$

$60 - 7r = 18$

$r = 6$

Coefficient = ${}^{15}C_6$

3. Sum of first 20 terms of the sequence 5, 11, 19, 29, ...

Ans. 3520

Sol. $T_n = an^2 + bn + c = 5$

$T_1 = a + b + c = 5$ .....(1)

$T_2 = 4a + 2b + c = 11$ .....(2)

$T_3 = 9a + 3b + c = 19$ .....(3)

(2) – (1) $\Rightarrow$ $3a + b = 6$

(3) – (2) $\Rightarrow$ $5a + b = 8$

$a = 1, b = 3, c = 1$

$T_n = n^2 + 3n + 1$

$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$

$S_{20} = \frac{20\times27\times41}{6} + \frac{3}{2}\times20\times21+20$

$= 70 \times 41 + 630 + 20 = 3520$

4. $(p\to q)\wedge(r\to q)$ is equivalent to

(1) $\sim p\wedge\sim r$ (2) $q\vee(\sim p\wedge\sim r)$ (3) $q\vee(\sim p\vee\sim r)$ (4) None of these

Ans. (3)

Sol. $(\sim p\vee q)\wedge(\sim R\vee Q)$

$= q\vee(\sim p\wedge\sim R)$

Ans. (B)

5. If $2x^y + 3y^x = 20$ then find $\frac{dy}{dx}$ at (2, 2)

Ans. $\frac{-3}{5}\ell n2$

Sol. $2x^y + 3y^x = 20$

$\Rightarrow 2.\ y.\ x^{y-1}.\ \frac{dy}{dx} + 3.y^x\left(\ell n y + \frac{x}{y}.\frac{dy}{dx}\right) = 0$

$\Rightarrow 2.\frac{dy}{dx}.y.x^{y-1} = \left(-3y^x.\ell n y\right)$

$\Rightarrow \frac{dy}{dx}\left[2y.x^{y-1} + 3xy^{x-1}\right] = -3y^x.\ell n y$

at x = 2, y = 2

$\Rightarrow \frac{dy}{dx}\left[2\times2\times2^1 + 3\times2\times2^1\right] = -3\times4\times\ell n2$

$\Rightarrow \frac{dy}{dx}\left[5\right] = -3\ell n^2 \Rightarrow \frac{dy}{dx} = \frac{-3}{5}\ell n2$

6. For two groups of 15 sizes each, mean and variance of first group is 12, 14 respectively, and second group has mean 14 and variance σ^2. If combined variance is 13 then find variance of second group?

(1) 9 (2) 11 (3) 10 (4) 12

Ans. (3)

Sol. $\frac{\sum x_i}{15} - (12)^2 = 14$

$\sum x_i^2 = 158\times15$

$\frac{\sum x_i}{15} = 12$

$\Sigma x_i = 180$

$\frac{\sum y_i}{15} = 14$

$\frac{\Sigma y_i}{15} = 14$

$\Sigma y_i = 15 \times 14$

Combined mean $= \frac{\Sigma x_i + \Sigma y_i}{30} = \frac{15\times12 + 15\times14}{30} = 13$

$\frac{\Sigma y_i^2 + \Sigma x_i^2}{30} - (13)^2 = 13$

$\frac{\Sigma y_i^2 + 158\times15}{30} = 13\times14$

$\Sigma y_i^2 = 30\times13\times14 - 158\times15$

$\sigma^2 = \frac{30\times13\times14 - 158\times15}{15} - (14)^2$

$\sigma^2 = 10$

7. A matrix of 2 × 2 satisfies $A^2 - I = 0$. If trace (A) = a and |A| = b then find $3a^2 + 4b^2$.

Ans. 4

Sol. Characteristic equation of 2 × 2 matrix

$A^2 - tr(A) \cdot A + |A| \cdot I = 0$

tr(A) = 0

|A| = 1

$3a^2 + 4b^2 = 4 \times 1 = 4$

8. Given $f(x) = [a + 13\sin x]$. Find the points of non-differentiability of f(x) in (0, π). Given 'a' is an integer

Ans. 25

Sol. f(x) = a + [13sinx]

When 13sin(x) = An integer = I(Let)

But $0 < \sin(x) \le 1$ hence, $13\sin(x) \in (0, 13]$

In $x \in (0, \pi)$ number of points where $13\sin(x) \in (0, 13]$ = 12 × 2 + 1 = 25

9. Find the number of ways of distributing 20 oranges among 3 children such that each gets atleast one orange

Ans. ${}^{19}C_2$

Sol. $x_1 + x_2 + x_3 = 20$

$x_i \ge 1$

$x_1' + x_2' + x_3' = 17$ (let $x_i' = x_i + 1$)

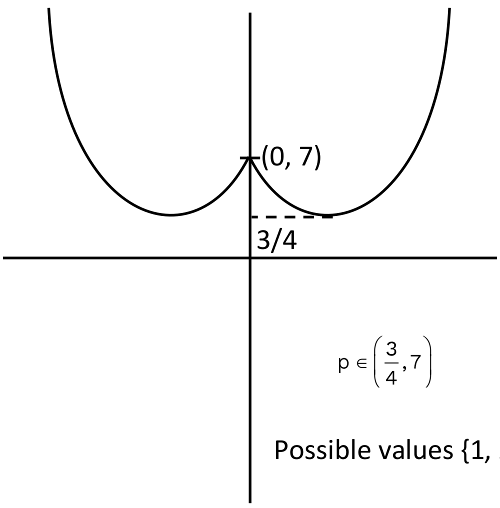
$x_i' \ge 0$

${}^{17+3-1}C_{3-1} = {}^{19}C_2$

10. $\left|x^2 - 5|x| + 7\right| = P \quad (P \in I)$ find number of possible values of "P" such that equation have 4 solutions.

Ans. 6

Sol.



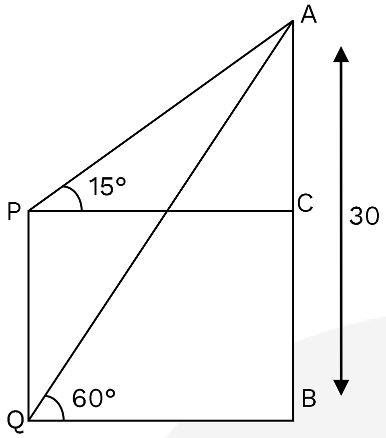
$p \in \left(\frac{3}{4}, 7\right)$

Possible values {1, 2,....., 6}

11. AB is building of height 30 m with angle of depression from top of building A to P. and Q is 15° and 60° respectively. A point C is on the same level on building with point P, then find the area of rectangle PCBQ?

Ans. $600\left(\sqrt{3}-1\right)$

Sol.



$\frac{30}{BQ}=\sqrt{3}\Rightarrow BQ=\frac{30}{\sqrt{3}}=10\sqrt{3}$

$\frac{AC}{10\sqrt{3}}=2-\sqrt{3}$

$AC=\left(2-\sqrt{3}\right)10\sqrt{3}$

Area = $10\sqrt{3}\left[60-20\sqrt{3}\right]=600\left(\sqrt{3}-1\right)$

12. Given that $f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$, find the value of $\int_1^2 18f(x)\,dx$.

Ans. $10\ell n\ 2-6$

Sol. $5f\left(4\right)+4f\left(\frac{1}{x}\right)=\left(\frac{1}{x}+3\right)\times 5$

$5f\left(\frac{1}{x}\right)+4f\left(x\right)=x+3\times 3$

$9f\left(x\right)=\frac{5}{x}-4x+3$

$f\left(x\right)=\frac{5}{9x}-\frac{4x}{9}+\frac{1}{3}$

$\int_1^2 18f\left(x\right)dx=1\int_1^2\left(\frac{1}{x}-8x+6\right)dx$

$=\left(10\ell n x-\frac{8x^2}{2}+6x\right)_1^2$

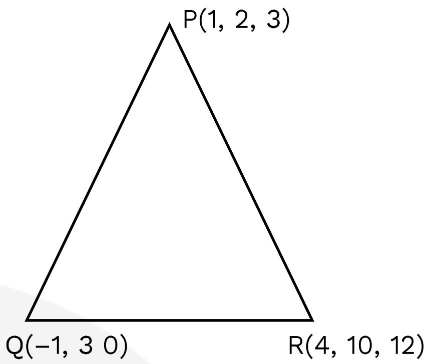
$=10\ell n2-6$

13. If the image of point P(1,2,3) about the plane 2x−y+3z = 2 is Q, then the area of triangle PQR. Where coordinates of R is (4,10,12).

Ans. $\frac{\sqrt{1531}}{2}$

Sol. Image formula → $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{2}=\frac{-2(ax_1+by_1+cz_1d)}{a^2+b^2+c^2}$

$\frac{x-1}{2}=\frac{x-y_1}{-1}=\frac{2-3}{3}=\frac{-2(2-2+9-2)}{4+1+9}$



⇒ x= −1

y = 3

z = 0

so, P(1, 2, 3)

Q(1, 2, 3) $\quad \overrightarrow{PQ}=-2i+J-3k$

R(4, 10, 12) $\quad \overrightarrow{PR}=3i+8j-3k$

Area if ΔOQR $=\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{QR}\right|$

$=\frac{1}{2}\left[i(9+24)-j(-18+9)+k(-16-3)\right]$

$=\frac{1}{2}\left[3\hat{i}+9\hat{j}-19\hat{k}\right]$

$=\frac{\sqrt{33^2+9^2+19^2}}{2}=\frac{\sqrt{1531}}{2}$

14. The sum of roots of $|x^2-8x+15|-2x+7=0$

(1) $11+\sqrt{3}$ (2) $11-\sqrt{3}$ (3) $9+\sqrt{3}$ (4) $9-\sqrt{3}$

Ans. (3)

Sol. (1) → $x\in(-\infty,3]\vee[5,\infty)$

(x − 3) (x − 5) − 2x + 7 = 0

⇒ $x^2-8x+15-2x+7=0$

⇒ $x^2-10x+22=0 \rightarrow x=\frac{10\pm\sqrt{100-88}}{2}$

$x=5\pm\sqrt{3}$

$x=5+\sqrt{3}$

(2) → x ∈ (3, 5)

−(x − 3) (x − 5) −2x +7 = 0

$-x^2+8x+15-2x+7=0$

⇒ $(x^2-6x+8)=0$

x = 4

Sum of roots $=5+\sqrt{3}+4$

$=9+\sqrt{3}$

15. let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}, \vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{d}$ is vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a}.\vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is

(1) 640 (2) 720 (3) 680 (4) 760

Ans. (2)

Sol. $d = \lambda(\bar{b} \times \bar{c})$

$$\lambda\begin{vmatrix} i & j & k \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = \lambda(\hat{i}(2) - \hat{j}(1) + \hat{k}(2))$$

$\Rightarrow \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$(\bar{a} \times \bar{d}) = (2\ 3\ 4) \times (4\ -2\ 4)$

$$\begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 4 & -2 & 4 \end{vmatrix} = \hat{i}(20) - \hat{j}(-8) + k(-16)$$

$20\hat{i} + 8\hat{j} - 16\hat{k}$

400 + 64 + 256

= 720

16. Let $a_1, a_2, a_3 \ldots, a_n$ are in arithmetic progression having common difference d. The value of $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ is:

Ans. 1

Sol. $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_5}} + \ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ is

$$\frac{\left(\sqrt{a_1}-\sqrt{a_2}\right)}{\left(\sqrt{a_1}-\sqrt{a_2}\right)\left(\sqrt{a_1}+\sqrt{a_2}\right)} = \left(\frac{\sqrt{a_1}-\sqrt{a_2}}{-d}\right)$$

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}.\frac{1}{d}\left(\sqrt{a_2}-\sqrt{a_1}-\sqrt{a_2}-\sqrt{a_1}\ldots\ldots\sqrt{a_n}-\sqrt{a_{n-1}}\right)$$

$$\lim_{n\to\infty}\frac{1}{\sqrt{nd}}\frac{\left(\sqrt{a_n}-\sqrt{a_1}\right)\left(\sqrt{a_n}+\sqrt{a_1}\right)}{\sqrt{a_n}+\sqrt{a_1}}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt{n}\sqrt{d}}\frac{a_1+(n-1)d-a_1}{\sqrt{a_n}-\sqrt{a_1}}$$

$$\lim_{n\to\infty}\frac{(n-1)\sqrt{d}}{\sqrt{n}\sqrt{a_1+(n-1)d}+\sqrt{a_1}}$$

$$\lim_{n\to\infty}\frac{\left(1-\frac{1}{n}\right)\sqrt{d}}{\sqrt{\frac{a_1(n-1)d}{\sqrt{n}}}+\frac{\sqrt{a_1}}{\sqrt{n}}} = \frac{\sqrt{d}}{\sqrt{d}+0} = 1$$

$$\sqrt{\frac{a_1}{n}+\frac{(n-1)d}{n}}$$

17. The ratio of terms of 5^{th} term form beginning and 5^{th} term from end is $\sqrt{6}:1$ in $\left(2^{\frac{1}{4}}+\frac{1}{3^{\frac{1}{4}}}\right)^n$ the value of n is:

Ans. 10

Sol. $T_5 = {}^nC_4\left(2^{\frac{1}{4}}\right)^{n-4}\left(\frac{1}{3^{\frac{1}{4}}}\right)^4$

5^{th} term from last $= {}^nC_4\left(\frac{1}{3^{\frac{1}{4}}}\right)^{n-4}\left(2^{\frac{1}{4}}\right)^4$

$\frac{2^{\frac{n-4}{4}}}{3}\cdot\frac{3^{\frac{n-4}{4}}}{2} = \frac{\sqrt{6}}{1}$

$3^{\frac{n-8}{4}}.2^{\frac{n-8}{4}} = 6^{\frac{1}{2}}$

$\frac{n-8}{4} = \frac{1}{2} \Rightarrow 2n - 16 = 4$

$2n = 20$

$n = 10$

18. If ${}^{2n}C_3 : {}^nC_3 = 10$, then $\frac{n^2+3n}{n^2-3n+4}$ is equal to

Ans. 2

Sol. if ${}^{2n}C_3 : {}^nC_3 = 10$, then $\frac{n^2+3n}{n^2-3n+4}$ is equal to.....

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$

$2.2\frac{(2n-1)}{(n-2)} = 10 \Rightarrow 4n-2 = 5n-10$

$\boxed{n = 8}$

$\frac{64+24}{64-24+4} = \frac{88}{44} = 2$

19. The integration $\int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$ is

(1) $\frac{x}{x\tan x+1} + \log|x\sin x + \cos x| + C$

(2) $\frac{x}{x\tan x+1} - \log|x\sin x + \cos x| + C$

(3) $-\frac{x^2}{x\tan x+1} + 2\log|x\sin x + \cos x| + C$

(4) $\frac{x^2}{x\tan x+1} + 2\log|x\sin x + \cos x| + C$

Ans. (3)

Sol. $\int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$

Apply by parts and substituting (x tanx + 1) = t to solve the integral we get

We get $I = \frac{-x^2}{x\tan x + 1} + \int \frac{2x\cos(x)}{x\sin x + \cos x}dx$

Now substituting xsinx + cosx =t we get $I = \frac{-x^2}{x\tan(x)+1}$ + 2ln(| xsinx + cosx |) + C