MATHEMATICS

1. In how many ways the letter of words **"MATHEMATICS"** can be arranged. Where C and S do not come together.

Ans. $\frac{11!}{2!2!2!}-\frac{10!}{2!2!2!}\times 2!$

Sol. $\frac{11!}{2!2!2!}$

"MATHEMATICS"

Total No. of Arrangements by toking all the letters of word. Where C and S do not come together

MATHEMATICS

$\frac{11!}{2!2!2!}-\frac{10!}{2!2!2!}\times 2!$

2. If set A{1, 2, 3, 4, 5, 6, 7} and relation is define

R = {(x, y)|x, y ∈ A × A, x + y = 7} then which option is correct.

(1) Equivalence relation

(2) Transverse but not symmetric

(3) Symmetric only

(4) Reflexive

Ans. (3)

Sol. R = {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

Symmetric only

3. In the binomial expansion of $\left(2x^2+\frac{1}{2x}\right)^{11}$ find the difference of coefficient of x^{10} and x^7.

Ans. (1716)

Sol. $T_{r+1}={}^{11}C_r\left(2x^2\right)^{11-r}\left(\frac{1}{2x}\right)^r$

$= {}^{11}C_r\, 2^{11-2r}\ x^{22-3r}$

22-3r = 10	22-3r = 7
R = 4	r = 5

$\left|{}^{11}C_4 2^3-{}^{11}C_5 2\right|=\left|2\left[\frac{4\times 44.10.9.8}{24}-\frac{11.10.9.8.7}{120}\right]\right|$

$=2\left[11.10.9.8\left[\frac{1}{6}-\frac{7}{120}\right]\right]$

$=\frac{2\times 11\times 10\times 9\times 8[13]}{120}$

= 22 × 6 × 13

= 1716

4. Integrate $\int\left[\left(\frac{x}{2}\right)^x-\left(\frac{2}{x}\right)^x\right]\left(\ln\left(\frac{2}{x}\right)-1\right)dx$

Ans. $\left(\frac{2}{x}\right)^x-\left(\frac{x}{2}\right)^x+C$

Sol. Let $\left(\frac{2}{x}\right)^x = t$

$x(\ln 2 - \ln x) = \ln t$

$$x\left(-\frac{1}{x}\right)+(\ln 2-\ln x)=\frac{1}{t}dt$$

$$(\ln 2 - \ln x - 1) = \frac{1}{t}dt$$

$$\therefore \int\left(t+\frac{1}{t}\right)\frac{1}{t}dt$$

$$\int dt+\int\frac{1}{t^2}dt$$

$$t-\frac{1}{t}+C$$

$$\left(\frac{2}{x}\right)^x-\left(\frac{x}{2}\right)^x+C$$

5. In the series 5, 8, 14, 23, 35, 50....................a_n is the nth term and S_n is the sum of 'n' terms then find the value of $S_{30} - a_{30}$?

Ans. 13720

Sol. $S_n = 5 + 8 + 14 + 23 + 35 + 50 + \ldots\ldots\ldots\ldots + T_n$

$S_n = \quad 5 + 8 + 14 + 23 + 35 + \ldots\ldots\ldots\ldots\ldots\ldots + T_{n-1} + T_n$

____

$a_n = 5 + (3 + 6 + 9 + 12 + 15 + \ldots\ldots\ldots\ldots + (T_n - T_{n-1}))$

$$a_n = 5+\frac{3.n(n-1)}{2}$$

$$= \frac{3n^2-3n+10}{2}$$

$$a_{30} = \frac{2700-90+10}{2}=\frac{2620}{2}=1310$$

$$S_{30} = \frac{3}{2}.\frac{30.31.61}{6}-\frac{3}{2}.\frac{30.31}{2}+\frac{10}{2}.30$$

$$S_{30} = \frac{3}{2}.31\left[365-45\right]+150$$

$$=\frac{3}{2}.31(320)+150$$

$$=(93)(160)+150$$

$= 14880 + 150$

$= 15030$

Hence, $S_{30} - a_{30} = 15030 - 1310$

$= 13720$

6. The mean and variance of 12 observation is $\frac{9}{2}$ and 4, later on two observations considered as 9 and 10, Instead of 7 and 14. Then the correct variance is $\frac{m}{n}$ (m and n are coprime). Then find m + n?

Ans. 311

Sol. $\frac{\sum xi}{12} = \frac{9}{2}$ $\qquad$ $\sum xi = 54$

But $\sum x_{new} = 56$ $\qquad$ new mean $\frac{56}{12} = \frac{14}{3}$

$\frac{\sum x_i^2}{12} - \left(\frac{81}{4}\right) = 4$ $\qquad$ $\sum x_{new}2 = 355$

$\frac{\sum x_i^2}{12} = \frac{97}{4}$

new variance

$= \frac{355}{12} - \left(\frac{14}{3}\right)^2$

$\sum x_1^2 = 291$ $\qquad$ $= \frac{355}{12} - \frac{196}{9} = \frac{843}{108} = \frac{281}{36}$

7. (p ∧~q) ∨ (~p) is equivalent to.

(1) p ∨ q $\qquad$ (2) p ∧ q $\qquad$ (3) ~(p ∧ q) $\qquad$ (4) ~(p ∨ q)

Ans. (3)

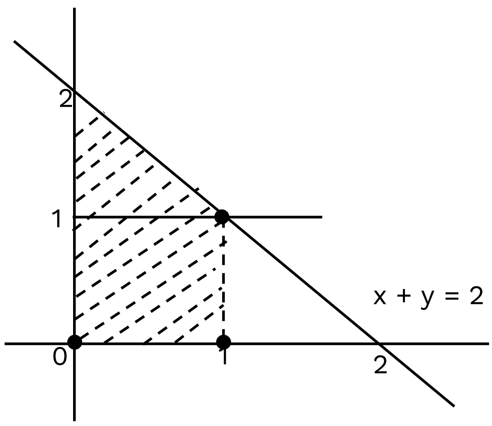
Sol. (~p ∨ q) ∧ (~p ∨ ~q)

t ∧ (p ∧ q)

= ~(p ∧ q)

8. Area enclosed by x + y = 2, y = 0, and $f(x) = \text{minimum}\left\{[x], x^2 + \frac{3}{4}\right\}$

Ans. $\frac{3}{2}$



Sol.

Min f(x) = [x]

Area = $1 + \frac{1}{2} \times 1 \times 1 = \frac{3}{2}$

9. If f : A → B where A = {a, b, c, d, e} and B = {1, 2, 3, 4} then find the number of such onto functions defined from A → B where f(a) ≠ 1 :

Ans. 180

Sol. Total no of onto functions = $\frac{5!}{2!(1!)^3} \times \frac{1}{3!} \times 4! = 240$

f(a) ≠ 1 no. of such functions are $240 \times \frac{3}{4}$

= 180

10. If $A = \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$ and $A^7 = \alpha^6 A + BI$ then find $\alpha^2 + \beta^2$?

Ans. 121

Sol. Ch. equation will be. $A^2 = 11A$

$A^2 - 11A + OI = 0$

$(A^2)^3 A = (11A)^3.A$

$= 11^3 A^4$

$= 11^3 (11A)^2$

$= 11^3 \times 121 A^2$

$A^7 = 11^6.A$

$A^7 = \left(A^2\right)^3.A$

$= (11 A)^3.A$

$= 11^3A^4$

$= 11^3 (11A)^2$

$= 11^3 11^2 (A)^2$

$= 11^6 A$

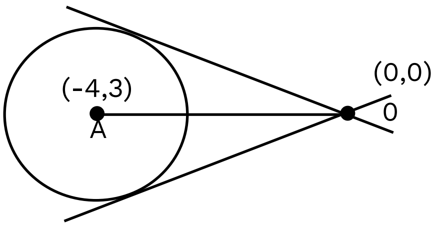
$\alpha^6 = 11^6$ $\qquad \alpha = 11$ $\qquad \beta = 0$

$\alpha^2 + \beta^2 = 121$

11. Two tangents OP and OQ are drawn from origin to the circle $x^2 + y^2 + 8x - 6y + 5 = 0$. Then circumcentre of triangle OPQ is.

(1) $\left(-2, \frac{3}{2}\right)$ (2) $\left(2, \frac{-3}{2}\right)$ (3) $\left(-2, \frac{-3}{2}\right)$ (4) $\left(2, \frac{3}{2}\right)$

Ans. (1)



Sol.

Circumcentre is midpoint of A0 = $\left(-2, \frac{3}{2}\right)$

12. Find the volume of tetrahedron ABCD with vector A(2, 1, 1), B(1, 2, 5), C(−2, −3, 5), D(1, −6, −7).

Ans. 0

Sol. Volume = $\left|\frac{1}{6}\begin{vmatrix} -1 & 1 & 4 \\ -4 & -4 & 4 \\ -1 & -7 & -8 \end{vmatrix}\right|$

$= \frac{-2}{3}\begin{vmatrix} 1 & 1 & 4 \\ 0 & -2 & -3 \\ 0 & -6 & -9 \end{vmatrix}$

$= \frac{-2}{3}\left(18 - 18\right) = 0$

13. $A = \{\theta, \theta \in (0, 2\pi), \frac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary$\}$ find the sum of elements in A.

Ans. Sum = 4π

Sol. $\left(\frac{1+2i\sin\theta}{1-i\sin\theta}\right) \times \left(\frac{1+i\sin\theta}{1+i\sin\theta}\right)$

$1-2\sin^2\theta = 0$

$\sin^2\theta = \frac{1}{\sqrt{2}}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\qquad$ Sum = 4π

14. If $\frac{1}{x}, \frac{1}{y}, \frac{1}{z} \rightarrow$ A.P and $x, \sqrt{2}y, z \rightarrow$ G.P and xy + yz + zx = $\frac{3}{\sqrt{2}}xyz$ then find $3(x + y + z)^2$.

Ans. 150

Sol. $2y^2 = xz,\ y = \frac{2xz}{x+z}$

$$xy + yz + zx = \frac{3}{\sqrt{2}}xyz$$

$$3xz = \frac{3}{\sqrt{2}}xyz$$

$$y = \sqrt{2}$$

$$xz = 4$$

$$y = \frac{8}{x+z}$$

$$x + z = 4\sqrt{2}$$

$$3\left(4\sqrt{2} + \sqrt{2}\right)^2 = 3 \times 50 = 150$$

15. If α, β are the roots of $ax^2 + bx + 1 = 0$ and $\lim\limits_{x \to \frac{1}{\propto}} \alpha \left[\frac{1 - cox\left(x^2 + bx + a\right)}{2(1 - \alpha x)^2}\right]^{\frac{1}{2}} = \frac{1}{k}\left|\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)\right|$ then find k.

Ans. 2

Sol. $ax^2 + bx + 1 = 0 \ \begin{cases} \alpha \\ \beta \end{cases}$ $\qquad x^2 + bx + a = 0 \ \begin{cases} \frac{1}{\alpha} \\ \frac{1}{\beta} \end{cases}$

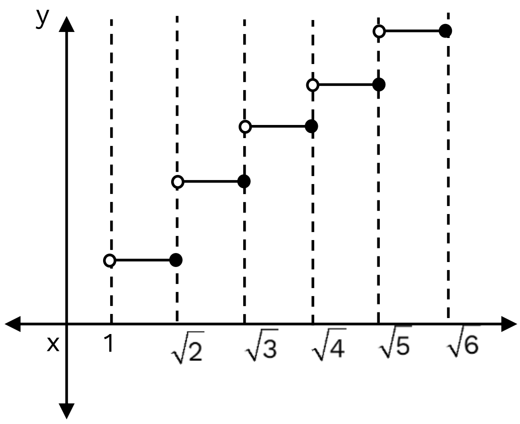
$$\lim_{x \to \frac{1}{\alpha}} \alpha \left(\frac{1 - \cos\left(x^2 + bx + a\right)}{2(1 - \alpha x)^2}\right) = \lim_{x \to \frac{1}{\alpha}} \alpha \left[\frac{1 - \cos\left(x^2 + bx + a\right)}{\left(x - \frac{1}{\alpha}\right)^2\left(x - \frac{1}{\beta}\right)^2} \times \frac{\left(x - \frac{1}{\alpha}\right)^2\left(x - \frac{1}{\beta}\right)^2}{2\alpha^2\left(x - \frac{1}{\alpha}\right)^2}\right]^{\frac{1}{2}}$$

$$\frac{\alpha}{2}\frac{\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)}{\alpha} = \frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)$$

k = 2

16. The value of $\int_0^{2.4} \left[x^2\right]dx$ is $\alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$ then $(\alpha + \beta + \gamma + \delta)$ is equal to________ .

Ans. 6



Sol.

$$\int_0^{2.4}\left[r^2\right]dx = \int_0^1 0dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{\sqrt{4}} 3dx + \int_{\sqrt{4}}^{\sqrt{5}} 4dx + \int_{\sqrt{5}}^{2.4} 4dx + \int_{\sqrt{5}}^{2.4} 5dx$$

$$= \left(\sqrt{2}-1\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(\sqrt{4}-\sqrt{3}\right) + 4\left(\sqrt{5}-\sqrt{4}\right) + 5\left(2.4-\sqrt{5}\right)$$

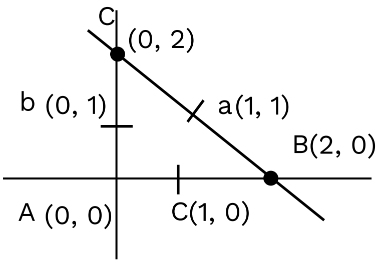
$$= 9-\sqrt{2}-\sqrt{3}-\sqrt{5}$$

$\therefore \alpha = 9, \beta = -1, \gamma = -1, \delta = -1$

$\Rightarrow \alpha + \beta + \gamma + \delta = 6$

17. The mid point of side of a triangle are (0, 1), (1, 0), (1, 1) whose incentre is D. A parabola $y^2 = 4ax$ passes through D whose focus is $\left(\alpha + \beta\sqrt{2}, 0\right)$, then $\frac{\beta^2}{\alpha}$ is:

Ans. ()



Sol.

$a = 2\sqrt{2}$

$b = 2$

$c = 2$

$$\text{incentre } I = \left(\frac{2\sqrt{2}.0 + 2.2 + 2.0}{2+2+2\sqrt{2}}, \frac{4}{2+2+2\sqrt{2}}\right)$$

$$I = \left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$$

$y^2 = 4ax$ passes through I

$$\left(\frac{4}{4+2\sqrt{2}}\right)^2 = 4a\left(\frac{4}{4+2\sqrt{2}}\right)$$

$$a = \frac{1}{4+2\sqrt{2}}$$

$$a = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

Focus = (a, 0)

$$\alpha.\frac{1}{2}, \beta = -\frac{1}{4} \Rightarrow \frac{\beta^2}{\alpha} = \frac{1}{8}$$

18. In probability distribution for discrete variable x = 0, 1, 2..............P(X = x) = $k(x+1)3^{-x}$. The probability of $P(x \geq 2)$ is equal to.

Ans. $\frac{7}{27}$

Sol. P(x = 0) + p(x = 1) + P(x = 2)+ __________ = 1

$$k(1) + k\left(\frac{2}{3}\right) + k\left(\frac{3}{3^2}\right) + \_\_\_\_\_\_\_\_\_\_ = 1$$

$$k\left\{1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{4^3}+\ldots\ldots\ldots\ldots\right\}=1$$

This is A.G.P = S

$$S=1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{4^3}+\ldots\ldots\ldots\ldots$$

$$\frac{S}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+\ldots\ldots\ldots\ldots$$

$$\frac{2}{3}S=1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots\ldots\ldots$$

$$\frac{2S}{3}=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$$

$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$$=1-\frac{4}{9}-\frac{4}{9}\left(\frac{2}{3}\right)$$

$$\Rightarrow \frac{7}{27}$$